Lossless and Sufficient $\Psi$ Invariant Decomposition of Deterministic Target

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Philosophy behind this work

- Radar polarimetry physically models the scattering process along the T.E. plane
- The S matrix decomposition would:
  - Model the degrees of freedom (DoF) of the received signal for target discrimination
  - Provide a physical interpretation of the parameter space for inversion studies
  - Present a maximum degree of invariance
  - Be easy to perform
- The faster decomposition is, the easier to understand would be
- The circular polarization basis is also related to photon’s polarization [S. R. Cloude PHD].
Previous work, invariant features selection between Cloude-Pottier ITD parameters and application to man-made targets


Eigenvector form of $T$: $A = SPAN \begin{bmatrix} P_1 & 0 & 0 \\ 0 & P_2 & 0 \\ 0 & 0 & P_3 \end{bmatrix} = U^H T U$, $U = U_P = \begin{bmatrix} u_{P-1} & u_{P-2} & u_{P-3} \end{bmatrix}$

$u_{P-i} = \begin{bmatrix} \cos(\alpha_P) \\ \sin(\alpha_P) \cos(\beta_P) e^{j\delta_P} \\ \sin(\alpha_P) \sin(\beta_P) e^{j\gamma_P} \end{bmatrix}$

$u_{P-i}^\Psi = R(\Psi) u_{P-i} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(2\Psi) & \sin(2\Psi) \\ 0 & -\sin(2\Psi) & \cos(2\Psi) \end{bmatrix} u_{P-i} \rightarrow$

$u_{P-i}^\Psi = \begin{bmatrix} \alpha_{P-i} \beta_{P-i} e^{j\gamma_{P-i}} \end{bmatrix}$

$\gamma_{P-i}, \delta_{P-i} \neq \delta_{P-i}$

- Eigenvector parameters (12), are not all $\Psi$-invariant in Pauli Basis.

- Eigenvalues $\Psi$-invariance has been demonstrated by E. Pottier and J. S. Lee, by calling $U^\Psi = R^\Psi U$ than $T^\Psi = U^\Psi T U^\Psi$.

$[SPAN \begin{bmatrix} P_1 & P_2 & P_3 \ \\ \alpha_1 & \alpha_2 \ \\ \alpha_3 \end{bmatrix}$ are $\Psi$-invariant, nevertheless $P_3 - \alpha_3$ are redundant and noisy, and SPAN is range dependent.

So we select: $v^{\\text{Mag}} = [P_1, \alpha_1]$

$\Psi - SPAN$ invariants

- Little circular shift, velocity insensitivity [Pal_2010A]

- Little, data format, focusing, insensitivity [Pal_2010B]
Main contributes detailed in this presentation:

1. Definition of lossless and sufficient $\Psi$-invariant target vector decomposition
2. Definition of all in-one metric for unsupervised classification of deterministic target
3. Relationship of equivalence between CTDs for coherent targets
4. Definition of a proper parameter for characterizing the degree of non-symmetry

Further contributes presented in the Poster Session:

1. Definition of a lossless and sufficient $\Psi$-invariant random target decomposition
2. Definition of a generalized unsupervised classification scheme
## Why Lossless and Sufficient $\Psi$-Invariant Target Decomposition

<table>
<thead>
<tr>
<th>Deterministic Target Decomposition (CTD)</th>
<th>Stazionary Target Single Observation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random Target</td>
<td>Random Target</td>
</tr>
<tr>
<td>Decomposition (ITD)</td>
<td>Multi-looked data</td>
</tr>
</tbody>
</table>

### Lossless Decomposition

<table>
<thead>
<tr>
<th>Lossless &amp; Sufficient Decomposition</th>
<th>Represents all the Signal N-Dof</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lossless &amp; Sufficient Deco.</td>
<td>Represents all the Signal N-Dof with a minimum set $M=N$.</td>
</tr>
<tr>
<td>$\Psi$-invariance special constraint</td>
<td>$S(\Psi,\theta,\Phi) \rightarrow S(\theta,\Phi)$</td>
</tr>
</tbody>
</table>

---

**Decomposition**

- N-Dof Observation Space
- M-Dof Decomposed Parameter Space

**Signal Reconstruction**

- Psi Orientation
- 3° Rotation around $Y^*$ axis
- Phi Azimuth
- 1° Rotation around $Z^*$ axis
- Eta Elevation
- 2° Rotation around $X^*$ axis

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**Definition of Target Aspect angles**

Several Euler’s angle representations shall exist:

\[
\begin{bmatrix}
  i_x'' & i_y'' & i_z''
\end{bmatrix} =
\begin{bmatrix}
  \cos(\psi) & 0 & \sin(\psi) \\
  0 & 1 & 0 \\
  -\sin(\psi) & 0 & \cos(\psi)
\end{bmatrix}
\begin{bmatrix}
  1 & 0 & 0 \\
  0 & \cos(\theta) & \sin(\theta) \\
  0 & -\sin(\theta) & \cos(\theta)
\end{bmatrix}
\begin{bmatrix}
  \cos(\phi) & \sin(\phi) & 0 \\
  -\sin(\phi) & \cos(\phi) & 0 \\
  0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  i_x & i_y & i_z
\end{bmatrix}.
\]

(2)

We seek a polarization basis, where the orientation effect is simply modelled.
Review of Deterministic Target Decomposition Theorems

1. Kennough CTD [1952]:
   \[ D = me^{2\rho} \begin{bmatrix} 1 & 0 \\ 0 & \tan^2(\gamma) \end{bmatrix} = U(\psi, \tau, v)^T_{aa} S_{hv} U(\psi, \tau, v)_{aa} \]

2. Cloude - Pauli basis scattering vector [1985]:
   \[ k_P = \begin{bmatrix} S_{hh} + S_{ve} \\ S_{vh} - S_{ve} \\ S_{hv} + S_{vh} \end{bmatrix} = \frac{1}{2} \text{tr} \{ S_{hv} B_i \} \]

3. Krogager CTD [1990]:
   \[ T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad D_{\Psi} = \begin{bmatrix} \cos(2\Psi) & \sin(2\Psi) \\ \sin(2\Psi) & -\cos(2\Psi) \end{bmatrix}, \quad H = \begin{bmatrix} 1 & \pm j \\ \pm j & -1 \end{bmatrix}. \]

   \[ S_{hv} = e^{j\varphi_S} \left( k_b T + e^{j\varphi_D} \left( k_D D_{\Psi} + k_e e^{-j2\Psi} H^\pm \right) \right) \]

4. Cameron CTD [1990]:
   \[ S_{Sym} = U(-\Psi, \tau = 0)^T \frac{e^{j\varphi_{Sym}}}{\sqrt{1+|z|^2}} \begin{bmatrix} 1 & 0 \\ 0 & z \end{bmatrix} U(-\Psi, \tau = 0) \]

Main attacks to the four well-known decompositions:

1. Kennough CTD is not a function for identical con-eigenvalues [Cam_1990], hard computation, difficult interpretation for ATR.

2. The \(|k_P|\) is \(\Psi\) dependent [Tou_2007].

3. Krogager vector is not made by orthogonal elements [Tou_2007].

4. Cameron model is optimal for symmetric target but is lossy for partially symmetric set.

Goal: seek lossless and sufficient \(\Psi\)-invariant target vector model in spinor form, easily providing target shape invariants.
Modelling $\Psi$-effect

Diagonal Polar Modelling of the $\Psi$-effect [Holm-Barnes 1988]:

1. In 1988 Holm and Barnes, modelled $\Psi$-rotation on Pauli’s target vectors:

$$U = U_P = \begin{bmatrix} u_{P-1} & u_{P-2} & u_{P-3} \end{bmatrix}, U_P^\Psi = R(\Psi)U_P, R(\Psi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(2\Psi) & \sin(2\Psi) \\ 0 & -\sin(2\Psi) & \cos(2\Psi) \end{bmatrix}$$

2. By computing the eigenvector of $R(\Psi)$, the diag. polar form in $I_{\perp}$ basis

was found: $D(\Psi) = U^{-1}R(\Psi)U, c_{\Psi}^{\psi} = \begin{bmatrix} e^{-j2(\Psi+\Psi_0)} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{j2(\Psi+\Psi_0)} \end{bmatrix} c_{\Psi}^{-i} = D(\Psi)c_{\Psi}^{-i}.$

The solutions are also coincident to the polarization of elemental packets of energy introducing: “The Particle Theory of Radar Target Scattering”
1° Contribute: Development of Lossless and Sufficient $\Psi$-Invariant Deterministic Target Decomposition


$$S_{\pm} = \begin{bmatrix} S_{ll} & S_{ll+} \\ S_{ll+} & S_{ll-} \end{bmatrix} = U^T S_{ij} U = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix} \begin{bmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix}$$

$$S_{ll} = \frac{s_{hh} + j s_{hv} + j s_{vh} + s_{vv}}{2}$$
$$S_{ll+} = \frac{s_{hh} - s_{hv} - j s_{vh} + s_{vv}}{2}$$
$$S_{ll-} = \frac{s_{hh} - s_{hv} + j s_{vh} - s_{vv}}{2}$$
$$S_{ll+1} = \frac{-s_{hh} + s_{hv} + j s_{vh} - s_{vv}}{2}$$

After reciprocization $S_{ll+} = S_{ll-}$, the matrix is projected into $c$:

$$c = \frac{\sqrt{2}}{2} \begin{bmatrix} S_{ll} + S_{ll-} \\ S_{ll+1} \end{bmatrix} = \sqrt{SPAN} \begin{bmatrix} \sin(\alpha_C) \cos(\beta_C) e^{j \gamma - 2(\psi_P + \psi_0)} \\ \cos(\alpha_C) e^{-j \gamma} \\ \sin(\alpha_P) \sin(\beta_C) e^{j \gamma + 2(\psi_P + \psi_0)} \end{bmatrix} e^{j \Phi}$$

$|c| \rightarrow \alpha_C = \alpha_P$
$$\beta_C = \beta_{Corr} - \lambda_0$$

- $EI = \sin(\alpha_C) \cos(\beta_C)^2 - \sin(\alpha_C) \sin(\beta_C)^2 = 4\pi Huygen/SPAN$ measures power of non-symmetry

$arg\{c\} \rightarrow \{ \Phi \}$

$\Psi_C$
$$Y_C$$

- $Y_C$ characterizes odd-even scattering phase difference, adds information for anisotropic targets $\alpha_P \sim \frac{\pi}{4}$. If $\mathbf{S}$ is symmetric $\rightarrow (\alpha_C, Y_C) \leftrightarrow z$
- $\Psi_C$ models orientation.
- $\Phi_C$ models absolute phase.

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Contribute definition of an all in one metric for Deterministic Target Identification

\[
c_{-\Psi N} = \frac{R(\overrightarrow{\Psi})c}{||c||}
\]

- \( t = \Delta = \text{Trihedral} \)
- \( t = i = \text{Cylinder} \)
- \( t = I = \text{Dipole} \)
- \( t = V = \text{Narrow-Diplane} \)
- \( t = +^+ = +\frac{\pi}{4} - \text{Device} \)
- \( t = +^- = -\frac{\pi}{4} - \text{Device} \)
- \( t = @ = R - \text{Helix} \)
- \( t = \zeta = L - \text{Helix} \)

The decided class in M.L. sense is given by \( c_t \) having the maximum degree of correlation:

\[
C_D = \text{argmax}_t \left\{ \left| (c_{u-\Psi N}^H)^\frac{H}{H} c_{t-\Psi N} \right| \right\}
\]
Contribute definition of an all in one metric for deterministic target identification

\[ c^{-\psi N} = \frac{R(-\psi) c}{||c||} \]

\( t = \Delta = \text{Trihedral} \)
\( t = i = \text{Cylinder} \)
\( t = l = \text{Dipole} \)
\( t = V = \text{Narrow - Diplane} \)
\( t = {+}^+ = \frac{\pi}{4} = \text{Device} \)
\( t = {+}^- = -\frac{\pi}{4} = \text{Device} \)
\( t = \oplus = R - \text{Helix} \)
\( t = \odot = L - \text{Helix} \)

The decided class in M.L. sense is given by \( c_t \) having the maximum degree of correlation:
\[ C_D = \arg\max_t \left\{ \left| \left( c_a^{-\psi N} \right)^{\text{T}} c_t^{-\psi N} \right| \right\} \]

**H-Fork**: \( \tau = 11^\circ, \psi = 45^\circ, \nu = 30^\circ, \gamma = 15^\circ \)
Contribute relationships between deterministic target decompositions: A) Symmetric target

Two classical definitions of Symmetric Target have been provided:

- Kennaugh-Huynen theory defines a target as symmetric if the maximum co-polarization is linear \( \tau = 0 \):
  \[
  S_{hv}^{sym} = U(-\Psi, \tau = 0)^T \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix} U(-\Psi, \tau = 0)
  \]  
  \[ \text{Eq. (1)} \]

- Using the circular polarization signatures, a target is symmetric if \(|S_{ll}| = |S_{ll,l}|\).

- The two definitions are linked, if \( \beta_C = \pi/4 \rightarrow |S_{ll}| = |S_{ll,l}| = k, \rightarrow \)
  \[
  S_{hv} = 2jke^{2j(\Psi)} - e^{-2j(\Psi)} e^{\Phi - \gamma} = -4ke^{-j\gamma + \Phi} \sin(\psi)
  \]  
  \[ \text{Eq. (2)} \]

by inspection of (2) applying \( \Psi = 0 \rightarrow S_{hv} = 0 \), the target is diagonalized, than it is symmetric according to Cameron definition. The relationship between \( \alpha_C - \gamma_C \) and complex Cameron's \( z \) parameter is:

\[
z = \frac{1 - \tan(\alpha_C)}{1 + \tan(\alpha_C)} e^{-2j\gamma_C}
\]  
\[ \text{Eq. (3)} \]

Concluding:

\[
S^\text{Cam}_D = \frac{1}{\sqrt{1+|z|^2}} \begin{bmatrix} 1 & 0 \\ 0 & z \end{bmatrix},
S^\text{Huy}_D = me^{j2\nu+2\nu} \begin{bmatrix} 1 & 0 \\ 0 & \tan^2(\gamma) e^{-4\nu} \end{bmatrix} \rightarrow \left\{ \begin{align*}
|z| &= \tan^2(\gamma) \\
\angle z &= -4\nu
\end{align*} \right.
\]

Touzi SVM parameters, [Tou, 2007] are also dependent on Huynen's:

\[
\tan(\alpha_S) e^{j\Phi_{\alpha_S}} = \frac{1 - \tan^2(\gamma)e^{-4\nu}}{1 + \tan^2(\gamma)e^{-4\nu}},
\]

whereas SSCM parameters are equivalent to Cameron's \( z \):

\( \alpha_C, \gamma \leftrightarrow z \leftrightarrow \gamma \nu \leftrightarrow \alpha_S \Phi_{\alpha_S} \leftrightarrow \chi_c \Psi_c \).

A symmetric target is represented by a unit 2D scattering vector in linear polarization basis,

by using this condition all the distance measures are equivalent!
3° Contribute relationships between deterministic target decompositions: B) Partially symmetric target

\[
S_{\text{III}} = \begin{bmatrix}
S_{\text{lh}} & S_{\text{li}} \\
S_{\text{li}} & S_{\text{ll}}
\end{bmatrix} = U^T S_{xy} U = \frac{1}{\sqrt{2}} \begin{bmatrix}
1 & j \\
j & 1
\end{bmatrix} \begin{bmatrix}
S_{\text{hh}} & S_{\text{he}} \\
S_{\text{eh}} & S_{\text{ee}}
\end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix}
1 & j \\
j & 1
\end{bmatrix}
\]

After reciprocization \( S_{\text{III}} \rightarrow S_{\text{III}} \), the matrix is projected into \( c \):

\[
c = \frac{\sqrt{2}}{2} \left( S_{\text{III}} + S_{\text{III}} \right) = \sqrt{\text{SPAN}} \begin{bmatrix}
\sin (\alpha_C) \cos (\beta_C) e^{i \cdot \phi C - 2(\theta_C + \psi_C)} \\
\cos (\alpha_C) e^{-i \cdot 2\phi C - 2(\theta_C + \psi_C)}
\end{bmatrix} e^{i \cdot \Phi C}
\]

\[|c| \rightarrow \begin{cases}
\alpha_C = \alpha_C \\
\beta_C = \beta_C \text{ rev. - rad}
\end{cases}
\]

- \( El = \sin (\alpha_C) \cos (\beta_C)^2 - \sin (\alpha_C)^2 \sin (\beta_C)^2 \)
- power of non-symmetry

\[
4 \ell_{\text{Huygen}} = \frac{m^2}{2 \cos^2 (\gamma)} \cos (2\gamma) \sin (2\tau) = \text{SPAN} \cdot El.
\]
### Experimental Results 1: extraction of significant parameters

<table>
<thead>
<tr>
<th>$\Delta$</th>
<th>$S_{\nu\nu}$</th>
<th>$S_{L_{\perp}}$</th>
<th>$\alpha_{C}$</th>
<th>$E_{l}$</th>
<th>$\Upsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{\sqrt{2}}$</td>
<td>$\begin{bmatrix} 1 &amp; 0 \ 0 &amp; 1 \end{bmatrix}$</td>
<td>$\frac{1}{\sqrt{2}} \begin{bmatrix} 0 &amp; j \ j &amp; 0 \end{bmatrix}$</td>
<td>0</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>$\frac{\sqrt{3}}{\sqrt{5}}$</td>
<td>$\begin{bmatrix} 1 &amp; 0 \ 0 &amp; \frac{1}{2} \end{bmatrix}$</td>
<td>$\frac{1}{\sqrt{10}} \begin{bmatrix} 1 &amp; 2j \ 2j &amp; -1 \end{bmatrix}$</td>
<td>18</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$I$</td>
<td>$\begin{bmatrix} 1 &amp; 0 \ 0 &amp; 0 \end{bmatrix}$</td>
<td>$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 &amp; j \ j &amp; -1 \end{bmatrix}$</td>
<td>45</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$++$</td>
<td>$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 &amp; 0 \ 0 &amp; j \end{bmatrix}$</td>
<td>$\frac{\sqrt{2}}{2} \begin{bmatrix} 1 &amp; -1 \ -1 &amp; -1 \end{bmatrix}$</td>
<td>45</td>
<td>0</td>
<td>45</td>
</tr>
<tr>
<td>$--$</td>
<td>$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 &amp; 0 \ 0 &amp; -j \end{bmatrix}$</td>
<td>$\frac{\sqrt{2}}{2} \begin{bmatrix} 1 &amp; 1 \ 1 &amp; -1 \end{bmatrix}$</td>
<td>45</td>
<td>0</td>
<td>-45</td>
</tr>
<tr>
<td>$V$</td>
<td>$\sqrt{\frac{3}{5}} \begin{bmatrix} 1 &amp; 0 \ 0 &amp; -\frac{1}{2} \end{bmatrix}$</td>
<td>$\frac{1}{\sqrt{10}} \begin{bmatrix} 2 &amp; -j \ -j &amp; -2 \end{bmatrix}$</td>
<td>72</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$L$</td>
<td>$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 &amp; 0 \ 0 &amp; -1 \end{bmatrix}$</td>
<td>$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 &amp; 0 \ 0 &amp; -1 \end{bmatrix}$</td>
<td>90</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 &amp; j \ j &amp; -1 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 1 &amp; 0 \ 0 &amp; 0 \end{bmatrix}$</td>
<td>90</td>
<td>-1</td>
<td>-22.5</td>
</tr>
<tr>
<td>$S$</td>
<td>$\frac{1}{2} \begin{bmatrix} 1 &amp; -j \ -j &amp; -1 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 0 &amp; 0 \ 0 &amp; 0 \end{bmatrix}$</td>
<td>90</td>
<td>1</td>
<td>22.5</td>
</tr>
</tbody>
</table>

Table I

Extracted parameter from classical scatterers.

<table>
<thead>
<tr>
<th>SPAN</th>
<th>$\alpha_{C}$</th>
<th>$E_{l}$</th>
<th>$\Upsilon$</th>
<th>$\Psi$</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1-N</td>
<td>5.3</td>
<td>27</td>
<td>4</td>
<td>38</td>
<td>6</td>
</tr>
<tr>
<td>T1-S</td>
<td>1</td>
<td>12</td>
<td>-1</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>T1-E</td>
<td>2</td>
<td>67</td>
<td>-8</td>
<td>-14</td>
<td>31</td>
</tr>
<tr>
<td>T1-W</td>
<td>4.7</td>
<td>73</td>
<td>20</td>
<td>-27</td>
<td>1</td>
</tr>
<tr>
<td>T2-N</td>
<td>23.2</td>
<td>24</td>
<td>3</td>
<td>42</td>
<td>3</td>
</tr>
<tr>
<td>T2-S</td>
<td>2.7</td>
<td>30</td>
<td>4</td>
<td>33</td>
<td>6</td>
</tr>
<tr>
<td>T2-E</td>
<td>2</td>
<td>66</td>
<td>-9</td>
<td>-15</td>
<td>30</td>
</tr>
<tr>
<td>T2-W</td>
<td>0.3</td>
<td>10</td>
<td>0</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

Table II

Decomposed target parameters of two experimental POL-ISAR images.
Experimental results 2: development of Unsupervised Classification Scheme for Point Target (SAR Vs ISAR)

Extension to random target

Eigenvector form of $T : \Lambda = \text{SPAN} \begin{bmatrix} P_1 & 0 & 0 \\ 0 & P_2 & 0 \\ 0 & 0 & P_3 \end{bmatrix} = U^H T U,$

Diagonal Polar Modelling of the $\Psi$-effect [Holm-Barnes 1988]:

1. In 1988 Holm and Barnes, modelled $\Psi$-rotation on Pauli’s target vectors:

$$U = U_P = \begin{bmatrix} u_{p-1} & u_{p-2} & u_{p-3} \end{bmatrix}, U^\Psi = R(\Psi) U_P R(\Psi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(2\Psi) & \sin(2\Psi) \\ 0 & -\sin(2\Psi) & \cos(2\Psi) \end{bmatrix}$$

2. By computing the eigenvector of $R(\Psi)$, the diagonal polar form in $H_1$ basis was found:

$$\textbf{D} (\Psi) = U^{-1} R(\Psi) U, c_{-1}^\Psi = \begin{bmatrix} e^{-i2(\Psi+\Psi_0)} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i2(\Psi+\Psi_0)} \end{bmatrix} c_{-1}$$

3. By studding deeply the eigenvector basis DoF we have found the Lossless & Sufficient $\Psi$-Invariant Random Target Decomposition: $R^H I, R^H = R_\epsilon F_\gamma R_\alpha R_\beta E_5 E_\gamma$

Than by making the union of (1) and (2), and by switching the rows of $U_C$ through an SU(3) matrix, is found:

$$u_{c-1} = \begin{bmatrix} \cos(\alpha) \\ \sin(\alpha) \cos(\beta) e^{i(\alpha-2\Psi+2\Psi)} \\ \sin(\alpha) \sin(\beta) e^{i(2\Psi+2\Psi)} \end{bmatrix}$$
CTD applied to the dominant eigenvector of G

0 Pauli Basis Image
New parameters:
1. Dominant eigenvector $\alpha_1$
2. Dominant eigenvector quarter wave detector $\Upsilon_1$
3. Dominant eigenvector elicity $E_1$
4. Dominant eigenvector orientation angle $\Psi_1$
Further Results Poster Session> Development of Unsupervised Classification Technique for Distributed Target
Further Results > Development of Unsupervised Classification Scheme for distributed targets > Application on Data Mining.

Processing Notes:
1) Flying Pass Time 21.5’
2) Decomposition Time (500 Euro Notebook) 19’
3) Class. Time 5’
Conclusion:
• The lossless and sufficient $\psi$-invariant decomposition concept has been introduced for deterministic target.
• Circular polarization SU(2) basis has shown unique mathematical-physical features.
• An all-in one $\psi$-invariant classification algorithm has been proposed.
• The relationships between the more popular CTDs has been also traced.

• Generalization of this approach for the analysis of random target is provided in the Poster Session this afternoon.
• For any question, mail to: palaric@gmail.com
• Thank you for the attention.