Maximum Likelihood Shift Estimation using High Resolution Polarimetric SAR Clutter Model

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New High Resolution SAR sensors

(a) ERS 1995-07-09

(b) TSX 2009-01-06

Context
- High Resolution Polarimetric SAR data
- Natural risk
- Flow modeling
1. Introduction
2. ML texture tracking
3. Sensor merging
4. Conclusions
The Argentière test site

Location

Grenoble
The Argentière test site

Available data

- Annual Displacement Measurement

2008-09-29 HH

Resolution: 1.5m × 0.9m (Azimuth × Range)
The Argentière test site

Available data

- **Annual Displacement Measurement**
- **2006**: a *corner reflector* setting

Resolution: 1.5$m \times 0.9$m (Azimuth × Range)
The Argentière test site

Available data

- **Annual Displacement Measurement**
- 2006: a *corner reflector* setting
- Since summer 2007: *continuous GPS measurements* on 3 distinct areas.
The Argentière test site

Available data

- Annual Displacement Measurement
- 2006: a corner reflector setting
- Since summer 2007: continuous GPS measurements on 3 distinct areas.
- May 2009: 4 additional corners.

⇒ a well monitored glacier.
Common principle
Common principle
Common principle
Common principle
Displacement estimation

Principle

- **Master/Slave** texture images
Displacement estimation

Principle

- **Master/Slave** texture images.
- A **similarity criterion** is measured between two texture samples $\tau_x$ and $\tau_y$ extracted respectively from master and slave images.
Displacement estimation

Principle

- **Master/Slave** texture images.
- A **similarity criterion** is measured between two texture samples $\tau_x$ and $\tau^i_y$ extracted respectively from master and slave images.
- The displacement corresponds to the coordinates of the **maximum** of the criterion:

$$\vec{V}_{ML} = \text{Argmax } i \left( \tau_x | \tau^i_y, \vec{V}_i \right).$$

$\Rightarrow$ How to define a suitable statistically-based similarity criterion?
SIRV model: texture extraction

High Res. PolSAR data

SIRV

τ

z

MODELING

ML Displacement Estimation

Vx

Vy

ML texture tracking

Sensor merging

Conclusion
High Resolution airborne and spaceborne PolSAR sensors:
Context

- **High Resolution** airborne and spaceborne PolSAR sensors:
  - **Small** number of backscatterers in each resolution cell.

Clutter

- Homogeneous Clutter
- Heterogeneous Clutter
High Resolution airborne and spaceborne PolSAR sensors:
- Small number of backscatterers in each resolution cell.
- Homogeneous hypothesis of the PolSAR clutter may be reconsidered.
SIRV model

Context

- **High Resolution** airborne and spaceborne PolSAR sensors:
  - **Small** number of backscatterers in each resolution cell.
  - **Homogeneous hypothesis** of the PolSAR clutter may be **reconsidered**.

⇒ **Heterogeneous clutter** model: Spherically Invariant Random Vector (SIRV).

Clutter

- Homogeneous Clutter
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High Resolution airborne and spaceborne PolSAR sensors:
- Small number of backscatterers in each resolution cell.
- Homogeneous hypothesis of the PolSAR clutter may be reconsidered.

⇒ Heterogeneous clutter model: Spherically Invariant Random Vector (SIRV).

<table>
<thead>
<tr>
<th>SIRV Definition</th>
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<tbody>
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<td>$k = \sqrt{\tau} , z$</td>
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- $\tau$: scalar texture parameter.
- $z$: polarimetric target vector (speckle).

$z \sim \mathcal{N}(0, [M])$ with $[M] = E\{zz^H\}$. 
ML texture estimator

For a covariance matrix \([M]\), the \textbf{texture parameter ML estimator} \(\tau\) is given as:

\[
\hat{\tau}_i = \frac{k_i^H [M]^{-1} k_i}{p}.
\]

Normalized covariance matrix ML estimator

The \textbf{normalized covariance matrix ML estimator} in the deterministic texture case, is the solution of:

\[
[M]_{ML} = f([M]_{ML}) = \frac{1}{N} \sum_{i=1}^{N} \frac{k_i k_i^H}{\hat{\tau}_i}.
\]

- \textbf{Existence} and \textbf{uniqueness}, up to a scalar factor, of the normalized covariance matrix Fixed Point estimator [Pasc-08].
Modeling
Fisher PDF

\[
p_{\tau}(\tau) = \mathcal{F}[m, \mathcal{L}, \mathcal{M}] = \frac{\Gamma(\mathcal{L} + \mathcal{M})}{\Gamma(\mathcal{L})\Gamma(\mathcal{M})} \frac{\mathcal{L}}{\mathcal{M}m} \left(1 + \frac{\mathcal{L}_T}{\mathcal{M}m}\right)^{-1 - \mathcal{L} + \mathcal{M}}
\]

with \( m \) the scale parameter and \( \mathcal{L} > 0 \) and \( \mathcal{M} > 0 \) the shape parameters.
τ modeling

Fisher PDF

\[ p_\tau(\tau) = \mathcal{F}[m, L, M] = \frac{\Gamma(L + M)}{\Gamma(L)\Gamma(M)} \frac{L}{Mm} \left( \frac{L\tau}{Mm} \right)^{L-1} \left( 1 + \frac{L\tau}{Mm} \right)^{L+M} \]

with \( m \) the scale parameter and \( L > 0 \) and \( M > 0 \) the shape parameters.

κ₂/κ₃ plane

Modeling classification on texture image \( \tau \) of the serac fall of the Argentière glacier. Log-cumulant estimation with a 19 × 19 pixels sliding window.
Similarity criteria

ML texture tracking

Sensor merging

Conclusion
Uncorrelated Texture between images

**Ratio $\alpha = \frac{\tau_x}{\tau_y}$ distribution of 2 uncorrelated textures**

$$p_\alpha(\alpha) = \int_0^\infty p_\tau(\alpha\tau_y) p_\tau(\tau_y) \tau_y d\tau_y.$$ 

**Likelihood:**

$$L(\vec{\nu}_i) = \sum_{j=1}^{k} \ln (p_\alpha(\alpha_j)) = k \ln \left( \frac{B(2\mathcal{L}, 2\mathcal{M})}{[B(\mathcal{L}, \mathcal{M})]^2} \right) - (\mathcal{M} + 1) \sum_{j=1}^{k} \ln \tau_{x_j}$$

$$+ \mathcal{M} \sum_{j=1}^{k} \ln \tau_{y_j}^{i} + \sum_{j=1}^{k} \ln \left( 2F_1 \left( \frac{\mathcal{L} + \mathcal{M}, 2\mathcal{M}}{2 (\mathcal{L} + \mathcal{M}); 1 - \frac{\tau_{y_j}^{i}}{\tau_{x_j}}} \right) \right).$$

$\mathcal{L}$ and $\mathcal{M}$ : shape parameters of Fisher PDFs.

$2F_1(\cdot, \cdot; \cdot; \cdot)$ : Gauss hypergeometric function

$B(\cdot, \cdot)$ : Euler Beta function ($B(z, w) = \Gamma(z)\Gamma(w)/\Gamma(z + w), \Re(z) \geq 0, \Re(w) \geq 0$).
Correlated texture between images

**Ratio** \( \alpha = \frac{\tau_x}{\tau_y} \) distribution of 2 correlated textures

\[
p_{\alpha}(\alpha) = \int_{0}^{\infty} \tau_{y} p_{\tau_x \tau_y}(\alpha \tau_y, \tau_y) \ d\tau_y.
\]

⇒ use of the bivariate Fisher PDF.

**Likelihood:**

\[
L(\vec{v}_i) = K + (\mathcal{L}_1 - 1) \sum_{j=1}^{k} \ln \tau_{x_j} + \mathcal{L}_2 \sum_{j=1}^{k} \tau_{y_j}^i - a \sum_{j=1}^{k} \ln \left( R_1 \tau_{x_j} + R_2 \tau_{y_j}^i \right)
\]

\[
+ \sum_{j=1}^{k} \ln \left( 2F_1(a, b; c; z) \right).
\]

with \( a = \mathcal{L}_1 + \mathcal{L}_2, b = \mathcal{M}_2 - \mathcal{M}_1, c = \mathcal{L}_1 + \mathcal{M}_2, z = \frac{1}{1 + \frac{R_2}{R_1} \frac{1}{\alpha}}, R_1 = \frac{\mathcal{L}_1}{\mathcal{M}_1 m_1}, R_2 = \frac{\mathcal{L}_2}{\mathcal{M}_2 m_2} \)

and \( K = k \mathcal{L}_1 \ln R_1 + k \mathcal{L}_2 \ln R_2 + k \ln \left( \frac{B(\mathcal{L}_1 + \mathcal{L}_2, \mathcal{M}_2)}{B(\mathcal{L}_1, \mathcal{M}_1) B(\mathcal{L}_2, \mathcal{L}_1 + \mathcal{M}_2)} \right) \)
Correlated texture between images

**Ratio** $\alpha = \frac{\tau_x}{\tau_y}$ distribution of 2 correlated textures

$$p_\alpha(\alpha) = \int_0^\infty \frac{\tau_y}{\alpha \tau_y} \cdot p_{\tau_x \tau_y}(\alpha \tau_y, \tau_y) \, d\tau_y.$$  

$\Rightarrow$ use of the bivariate Fisher PDF.

**Likelihood:**

$$L(\vec{V}_i) = K + (\mathcal{L}_1 - 1) \sum_{j=1}^k \ln \tau_{x_j} + \mathcal{L}_2 \sum_{j=1}^k \tau_{y_j}^i - a \sum_{j=1}^k \ln \left( R_1 \tau_{x_j} + R_2 \tau_{y_j}^i \right)$$

$$+ \sum_{j=1}^k \ln \left( \frac{\mathcal{B}(\mathcal{L}_1 + \mathcal{L}_2, \mathcal{M}_2)}{\mathcal{B}(\mathcal{L}_1, \mathcal{M}_1)} \frac{\mathcal{B}(\mathcal{L}_2, \mathcal{L}_1 + \mathcal{M}_2)}{\mathcal{B}(\mathcal{L}_2, \mathcal{L}_1 + \mathcal{M}_2)} \right)$$

with $a = \mathcal{L}_1 + \mathcal{L}_2$, $b = \mathcal{M}_2 - \mathcal{M}_1$, $c = \mathcal{L}_1 + \mathcal{M}_2$, $z = \frac{1}{1 + \frac{R_2}{R_1} \frac{1}{\alpha}}$, $R_1 = \frac{\mathcal{L}_1}{\mathcal{M}_1 m_1}$, $R_2 = \frac{\mathcal{L}_2}{\mathcal{M}_2 m_2}$

and $K = k \mathcal{L}_1 \ln R_1 + k \mathcal{L}_2 \ln R_2 + k \ln \left( \frac{B(\mathcal{L}_1 + \mathcal{L}_2, \mathcal{M}_2)}{B(\mathcal{L}_1, \mathcal{M}_1) B(\mathcal{L}_2, \mathcal{L}_1 + \mathcal{M}_2)} \right)$
Various HR PolSAR sensors  $\rightarrow$ Sensor merging.
Various HR PolSAR sensors \( \implies \) Sensor merging.

**Preliminary hypothesis:** SAR acquisitions are synchronised.
The DEM is the common referential \[\Rightarrow\] LUT must be computed for each sensor.
The DEM is the common referential $\implies$ LUT must be computed for each sensor.

The similarity criteria $\sim$ Likelihood function

$$L_{ALL}(\vec{V}_i) = \sum_{j=1}^{k} \frac{1}{n_j} L_j(\vec{V}_i).$$
The DEM is the common referential \( \implies \) LUT must be computed for each sensor.

### Similarity criteria ~ Likelihood function

\[
L_{\text{ALL}}(\vec{v}_i) = \sum_{j=1}^{k} \frac{1}{n_j} L_j(\vec{v}_i).
\]
Results

Detection surfaces for motionless areas.

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RS2 and TSX detection surfaces: noisy. The merged case: sharp.

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Quality factor

\[ Q = \frac{\max_i(L(\vec{v}_i)) - \text{mean}_i(L(\vec{v}_i))}{\text{mean}_i(L(\vec{v}_i)) - \min_i(L(\vec{v}_i))} \]
Quality factor $Q$ computed on the whole Argentière glacier.

- NCC factor: low on the main part of the glacier.
Results

Quality factor $Q$ computed on the whole Argentière glacier.

- NCC factor: low on the main part of the glacier.
- Some border effects.
- Merged case: $Q$ is higher than without merging cases.
Conclusion and discussion

Generalization of the ML tracking method:
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- Multi-temporal and multi-frequency texture tracking.
Conclusion and discussion

Generalization of the ML tracking method:

- Multi-temporal and multi-frequency texture tracking.
- HR PolSAR clutter model.
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**Further works:**

- Removing acquisition synchronisation constraint.
- Texture parameters estimation.

⇒ Further results are shown on my poster.